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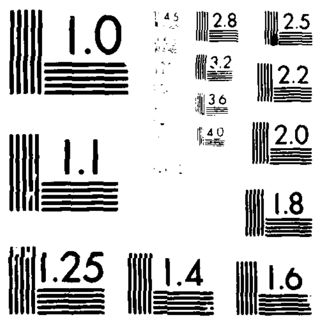

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MATHEMATICAL THEORY OF LAMINAR COMBUSTION. V.

Unsteady Burning of a Linear Condensate .

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Chapter V

Unsteady Burning of a Linear Condensate

1. The Problem

Chapter IV was concerned with the steady combustion of the gases produced by vaporization of a linear condensate at its surface, whether through pyrolysis, sublimation or evaporation. The results were characterized by response curves of burning rate versus pressure (represented by the Damköhler number). If the applied pressure varies in time, then so also must the burning rate; it is the nature of their dependence that is examined in the present chapter.

The effect of variations in pressure on solid pyrolysis has received a considerable amount of attention because of its relevance to the stability of solid-propellant rocket motors. The essential idea is that acoustic waves bouncing around the combustion chamber will impinge on the propellant surface and thereby generate fluctuations in the burning rate. These fluctuations will affect the reflected wave which, it is argued, can have a larger amplitude than the incident wave. If so, the transfer of energy (provided it is greater than losses through dissipation and other mechanisms) implies instability.

The response of a burning condensate (solid or liquid) to an impinging acoustic wave will be the focus of our discussion. Mathematically we must deal with the disturbance of a steady field containing large gradients, so that a frontal attack on the governing equations is not feasible. Six regions can be distinguished: condensate, preheat zone, flame, burnt gas, entropy zone and far field; so that, without rational approximation, the discussion soon disappears into numerical and/or ad hoc analysis. There

Foreward

This report is Chapter V of the twelve in a forthcoming research monograph on the mathematical theory of laminar combustion. Chapters I-IV originally appeared as Technical Reports Nos. 77, 80, 82 & 85; these were later extensively revised and then issued as Technical Summary Reports No's 1803, 1818, 1819 & 1888 of the Mathematics Research Center, University of Wisconsin-Madison. References to I-IV mean the MRC reports.

Contents

are three parameters other than θ^{-1} whose smallness can be exploited, namely the Mach number, the ratio of characteristic time in the gas to that in the condensate and the ratio of densities. Low Mach number enables the combustion approximation (with its spatial constancy of pressure) to be used in the combustion regions, namely preheat zone, flame and burnt gas; small time ratio ensures the gas phase responds much more rapidly than the condensate, so that the three combustion regions are quasi-steady if the acoustic frequency is sufficiently small (a restriction of no practical significance); and small density ratio implies that fluctuations in the location of the surface of the condensate may be neglected. The activation-energy asymptotics, needed for an analytical description of the combustion regions, determine how small the acoustic frequency must be: the resulting changes in flame temperature (due entirely to fluctuations in the condensate) must be $O(\theta^{-1})$.

Only adiabatic burning will be considered; no work on heat loss has so far been reported. The first task is to determine the response of the burning rate to a general (small) change in pressure level of the combustion regions, without regard to its cause. Sec. 2 determines the response for solid pyrolysis and Sec. 3 that for liquid evaporation. The incident acoustic wave excites such a response, and the response induces a reflected wave, through the intermediary of an entropy zone which converts isentropic conditions in the far field into isothermal ones in the near field (Sec. 4). Conditions for the reflected wave to be stronger than the incident wave are developed in Sec. 5.

Our analysis also enables stability characteristics to be discussed, at least on the time scale adopted, by holding the pressure level fixed

(i.e. eliminating the incident acoustic wave). The stability analysis of Ch. III is thereby extended to what may be called anchored flames. The extension continues in Sec. 6 to a flameholder, where the combustion field need no longer be taken quasi-steady but nevertheless still varies on a long time scale. Only in this last section, where the combustion field is truly unsteady, does the Lewis number play a role.

In view of the large number of small parameters we shall eschew writing formal expansions. Instead the sympol $<<$ will be used to indicate that only the leading term in the corresponding parameter is being considered.

2. Solid Pyrolysis

The unsteadiness will be treated as the perturbation of a steady state with Damköhler number D_o. Then, as in Sec. III.3, the burning rate M_o in the steady state will be taken as the mass flux on which units are based; according to the result (III.16), we have

$$M_o = \sqrt{2LD_o}\, T_\infty^2 \exp(-\theta/2\, T_\infty)/\theta \tag{1}$$

since none of the products of the gaseous reaction is produced or absorbed at the surface of the condensate $(J_s = 1)$. The flame temperature T_∞ is to be calculated from the formula (IV.3).

The smallness of the Mach number ensures that the combustion approximation is valid. If, in addition, the fluctuations in the applied pressure are not too rapid then the combustion is quasi-steady and the analysis of Ch. II is applicable. More precisely, if ω is a characteristic frequency of the fluctuations (e.g. the frequency of an impinging acoustic wave) then the time ω^{-1} should be long compared to the response time $\rho_c\lambda/c_pM_o^2$ of the gas phase, determined by diffusion. In short, we require

$$\omega << c_p M_o^2/\rho_c \lambda \quad , \tag{2}$$

a restriction of no practical consequence since the diffusion time is always very small. The same formula (1), with M_o and D_o replaced by M and D, therefore holds for the burning rate, where now both D and T_∞ are functions of time, the former prescribed and the latter to be determined from an analysis of heat transfer in the condensate and the nature of the vaporization.

This conclusion, which is not restricted to solid pyrolysis, was reached by Denison and Baum (1961) using ad hoc arguments which Williams (1973) later replaced with rational asymptotic analysis. However, none of these authors noted that (in general) a further restriction of ω is required if the Mach number is to remain sufficiently small for the combustion approximation to apply. In our formulation the restriction comes from allowing the flame temperature to be perturbed only by $O(\theta^{-1})$ amounts, so that the change in burning rate is $O(1)$. More precisely,

$$M = D^{1/2} e^{\phi_\infty/2} \tag{3}$$

where $T_\infty^2 \phi_\infty/\theta$ is the perturbation (T_∞ now being the unperturbed flame temperature) and M, D are measured in units of M_o, D_o. [The formula follows from the result (III.16).]

Now this perturbation is due to fluctuations of temperature in the condensate, and it may be determined by calculating the total change in enthalpy they produce up to the surface. In general, slow variations on the time scale $\hat{c}_p M_o^2/\hat{\rho}\hat{\lambda}$ of the condensate are needed if the result is to be $O(\theta^{-1})$, i.e.

$$\omega \sim \hat{c}_p M_o^2/\hat{\rho}\hat{\lambda}\theta \quad . \tag{4}$$

Note that, since the condensate is much denser and more conductive than the gas, i.e.

$$\rho_c/\hat{\rho} \ll 1 \quad \text{and} \quad \lambda/\hat{\lambda} \ll 1 \quad , \tag{5}$$

the change in enthalpy in the gas phase is negligible, i.e. in response to changing conditions the gas (unlike the condensate) has no inertia. Thus the restriction (4) implies

$$\omega \ll c_p M_o^2/\rho_c \lambda\theta \quad , \tag{6}$$

whereas rough equality would have to hold for there to be a perturbation. [Note that the condition (2) is satisfied a fortiori.] In special circumstances there is an alternative to slow variation, to which we shall come later.

To calculate the change in enthalpy we consider the dimensionless temperature equation

$$\partial T/\partial t + M\, \partial T/\partial x - \partial^2 T/\partial x^2 = 0 \tag{7}$$

in the condensate. Here $\hat{\rho}\hat{\lambda}/\hat{c}_p M_o^2$ and $\hat{\lambda}/\hat{c}_p M_o$ have been taken as units of time and length, while T is still referred to Q/c_p. Since temporal variations occur on the long-time scale

$$\tau = t/\theta \quad , \tag{8}$$

the term $\partial T/\partial t = \theta^{-1}\partial T/\partial\tau$, which represents local storage of enthalpy,

is seen to be a perturbation. If it is neglected, the leading approximation

$$T = T_{-\infty} + (T_s - T_{-\infty})e^{Mx} \tag{9}$$

is obtained, where M varies with τ. The analysis so far is valid whatever kind of vaporization occurs at the surface of the condensate; for solid pyrolysis, T_s is connected to M by the law (IV.5), which now reads

$$M = (k/M_o)T_s \exp(-\hat{\theta}/T_s) \ . \tag{10}$$

From the leading approximation we can calculate the neglected perturbation term, which represents a heat source/sink in equation (7), and hence the perturbation in flame temperature. We find

$$M\phi_\infty = -\int_{-\infty}^{o} \frac{\partial T}{\partial \tau}\, dx = -(p+q)M^{-2}dM/d\tau \quad , \tag{11}$$

where

$$p = T_s^2/(\hat{\theta}+T_s) > 0 \quad \text{and} \quad q = T_{-\infty} - T_s \quad , \tag{12}$$

so that, from the result (3), the equation connecting the variations in M and D is

$$(p+q)dM/d\tau = M^3 \ell n(D/M^2) \ . \tag{13}$$

The steady state $M = D = 1$ clearly satisfies the last equation; and we are mainly concerned with relating the small disturbances of M and D

caused by an acoustic wave. However the equation may also be used to investigate the inherent stability of the steady rate, i.e. the way M changes when D is held fixed at 1. We see that when M is disturbed from 1, either up or down, it is driven further away whenever

$$p+q < 0, \text{ i.e. } T_{-\infty} < \hat{\theta}T_s/(\hat{\theta}+T_s) \ . \tag{14}$$

The condition is difficult to interpret because T_s itself depends on M; but certainly there is linear instability if it is satisfied by the value of T_s corresponding to $M = 1$ in the pyrolysis law (10). This instability was predicted by Denison and Baum but it has apparently never been observed. Their condition, after extraneous terms are eliminated by letting $\theta \to \infty$, is actually

$$T_{-\infty} < \hat{\theta}T_s/(\hat{\theta}-T_s) \tag{15}$$

because their pyrolysis law lacks the factor T_s. Figure 1 shows the stability boundaries for a factor T_s^α with $\alpha = 0$, 1/2 and 1.

For a general pyrolysis law $M(T_s)$ the right side of the inequality becomes $T_s - MdT_s/dM$, and it is conceivable that no propellant satisfies such a condition. The procedure for checking a given $T_{-\infty}$ would be to compute the burning rate (1) for the assumed D_o and then determine T_s and dT_s/dM for $M = 1$ from the pyrolysis law.

For oscillations about the steady state forced by an acoustic wave, we write

$$M = 1+me^{i\tilde{\omega}\tau} \ , \quad D = 1+de^{i\tilde{\omega}\tau} \text{ with } \tilde{\omega} = \theta\omega \tag{16}$$

and linearize. Then

$$m = cd \quad \text{with} \quad c^{-1} = 2 + i\tilde{\omega}(p+q) \tag{17}$$

where T_s now has its value for $M = 1$. This relation determines the reflected wave in the far field, as we shall see in Sec. 4. [The instability result above is obtained by requiring c^{-1} to vanish.]

An alternative way for changes in flame temperature to be $O(\theta^{-1})$ is for the temperature in the solid to fluctuate by that amount everywhere. The fluctuations can then be on the scale of t, rather than τ, so that we are faced with solving the full heat equation (7) with M an arbitrary function of t. A general analysis would therefore be quite complicated; accordingly just the case of interest will be considered, namely small departures from the steady state.

When

$$M = 1 + me^{i\omega t} \quad \text{and} \quad D = 1 + de^{i\omega t} \quad \text{with} \quad m,\, d \ll 1 \tag{18}$$

the temperature in the solid has a steady component, given by the distribution (9) with $M = 1$ and T_s the corresponding surface temperature, and a fluctuating component

$$m[(-iq/\omega)\exp(x+i\omega t) + (p+iq/\omega)\exp(\kappa x+i\omega t)] \quad \text{with} \quad \kappa = (1+\sqrt{1+4i\omega})/2 \ . \tag{19}$$

The two contributions to this result are due to fluctuations in M within the solid and at its surface respectively. We now find

$$\theta^{-1}\phi_\infty = -\int_{-\infty}^{0} \frac{\partial T}{\partial t}\, dx = (1-\kappa)(p+q/\kappa)me^{i\omega t} \ , \tag{20}$$

in place of the result (11), so that

$$m = cd \quad \text{with} \quad c^{-1} = 2 - \theta(1-\kappa)(p+q/\kappa) \ , \tag{21}$$

which reduces to the previous response (17) for ω small but is quite different otherwise, as we shall see in Sec. 4.

The inherent stability of the steady state is determined by setting $c^{-1} = 0$. Then

$$P\kappa^2 + (Q-P+2)\kappa - Q \ = \ 0 \quad \text{with} \quad P = \theta p, \ \ Q = \theta q \ ; \tag{22}$$

for instability the parameter values P, Q must make the real part of $i\omega = \kappa(\kappa-1)$ positive for a κ whose real part is positive. Fig. 2 shows the region of instability in the P, Q-plane; its boundary asymptotes $P + Q = -6$, in accord with the previous result (14) for P, Q large.

For consistency, we must have

$$p,q \ = \ O(\theta^{-1}) \ , \tag{23}$$

which certainly requires both T_s and $T_{-\infty}$ to be of the same order. Such small temperatures do not mean that the condensate is cold, but only that the corresponding enthalpies are small compared to the heat of combustion, which is used as unit. The requirement (IV.6) for the steady state does however mean that T_∞, as calculated from the formula (IV.3), must be no greater than 1 (i.e. the pyrolysis must be endothermic).

3. Liquid Evaporation

Liquid propellants have also been used in rocket motors, though not in the manner envisaged here. A better motivation, if one is needed, is

a solid propellant which first liquefies at its surface and then vaporizes. If attention is again focused on adiabatic combustion, the analysis follows that in the last section until the pyrolysis law is reached, when the Clausius-Clapeyron relation (IV.21) is used instead, i.e. we write

$$D = kT_s^{\beta}(T_\infty - T_s)^{-1}\exp(-\hat{\theta}/T_s) \quad . \tag{24}$$

Here we have taken the Damköhler number proportional to $\bar{p}_c$ in accordance with the formula (I.59) for a first-order reaction (otherwise a power of D is required); and various parameters have been absorbed into k.

Since T_s is now a function of D, the latter enters into the temperature distribution (9) in the liquid, so that the formula (11) for the change in flame temperature becomes

$$M\phi_\infty = -(qM^{-2}dM/d\tau + pM^{-1}D^{-1}dD/d\tau) \tag{25}$$

where now

$$p = T_s^2(T_\infty - T_s)/[T_s^2 + (\beta T_s + \hat{\theta})(T_\infty - T_s)] > 0 \quad . \tag{26}$$

The equation connecting the variations in M and D is therefore

$$qM^{-1}dM/d\tau + pD^{-1}dD/d\tau = M^2 \ln(D/M^2) \quad .$$

In particular, there is inherent instability $(D \equiv 1)$ when q is negative, i.e. for

$$T_{-\infty} < T_s \quad . \tag{27}$$

Note that this instability (which has not been reported before) is independent of the precise form of the vaporization law at the surface: it occurs whenever the surface temperature is determined by the surface pressure.

For general small departures (18) from the steady state we find

$$m = cd \quad \text{with} \quad c = (1-i\tilde{\omega}p)/(2+i\tilde{\omega}q) \quad . \tag{28}$$

This result will be needed when we come to acoustic response in Sec. 5. When p and q are $O(\theta^{-1})$ higher frequencies ω can be considered, as for solid pyrolysis. The fluctuating component (19) in the condensate now becomes

$$(-iqm/\omega)\exp(x+i\omega t) + (pd+iqm/\omega)\exp(\kappa x+i\omega t) \ ,$$

where p has its new definition (26), so that

$$m = cd \quad \text{with} \quad c = [1+\theta(1-\kappa)p]/[2-\theta(1-\kappa)q/\kappa] \quad . \tag{29}$$

When D is held fixed at 1, i.e. $d = 0$, the denominator of c must vanish, so that $\kappa = Q/(2+Q)$. Instability requires $\kappa > 1$, i.e.

$$Q < -2 \quad \text{with} \quad Q = \theta q \ , \tag{30}$$

a condition in accord with the earlier result (27) for Q large.

4. Response to an Impinging Acoustic Wave

Analysis of the combustion zone provides, amongst other things, a complete description of the state of the burnt gas behind the flame sheet in terms of the time-dependent, but spatially uniform, pressure there.

Conditions are essentially isothermal with the velocity v_∞ determined by the instantaneous value of M.

The varying pressure is generated by some source far from the flame; we shall confine ourselves to acoustic waves travelling normal to the surface of the condensate, which immediately raises a problem. The far field is isentropic when the near field is isothermal, so that the two cannot be matched. In other words, it is not simply a matter of evaluating the pressure in the far field as the near field, represented by a single point, is approached. An "entropy" layer is needed to transform conditions from isentropic to isothermal in a distance small on the scale of the far field but large on that of the near field.

In the chemistry-free region beyond the flame sheet, the equations with which we have to deal are those of a compressible, heat-conducting, viscous fluid, i.e. the dimensional form of equations (I.53, 55, 56) with pressure and viscous terms restored in the energy balance, with $\Omega = 0$ and with the perfect-gas law reinstated. Linearized about the steady state ρ_∞, v_∞, $\bar{p}_c$, T_∞ they read

$$\partial\rho/\partial t + v_\infty \partial\rho/\partial x + \rho_\infty \partial v/\partial x = 0 , \tag{31}$$

$$\rho_\infty(\partial v/\partial t + v_\infty \partial v/\partial x) = -\partial p/\partial x + (4\kappa/3)\partial^2 v/\partial x^2 , \tag{32}$$

$$\rho_\infty c_p(\partial T/\partial t + v_\infty \partial T/\partial x) - \lambda \partial^2 T/\partial x^2 = \partial p/\partial t + v_\infty \partial p/\partial x . \tag{33}$$

It is natural to take ρ_∞, $a_\infty = \sqrt{\gamma R T_\infty/m}$, $\bar{p}_c = \rho_\infty a_\infty^2/\gamma$ and T_∞ as units of density, velocity, pressure and temperature in such a combustion-free region. Length will be referred to a distance ℓ to be specified later

and time to ℓ/a_∞. Then, for disturbances proportional to $e^{i\omega t}$, the dimensionless equations are

$$i\omega\rho + m_\infty \partial\rho/\partial x + \partial v/\partial x = 0 \ , \quad i\omega v + m_\infty \partial v/\partial x = -\gamma^{-1}\partial p/\partial x + \kappa_\ell \partial^2 v/\partial x^2 \ , \tag{34}$$

$$i\omega T + m_\infty \partial T/\partial x - \lambda_\ell \partial^2 T/\partial x^2 = (\gamma-1)(i\omega p + m_\infty \partial p/\partial x)/\gamma \ , \quad p = T + \rho \tag{35}$$

where

$$\kappa_\ell = 4\kappa/3\rho_\infty a_\infty \ell \quad \text{and} \quad \lambda_\ell = \lambda/c_p \rho_\infty a_\infty \ell \ . \tag{36}$$

Solution of these equations determines the dimensional density, velocity, pressure and temperature as

$$\rho_\infty(1+\rho e^{i\omega t}) \ , \quad v_\infty + a_\infty v e^{i\omega t} \ , \quad \bar{p}_c(1+p e^{i\omega t}) \quad \text{and} \quad T_\infty(1+T e^{i\omega t}) \tag{37}$$

respectively.

These equations, being linear with constant coefficients, can be solved exactly and the solution used to trace the transition from isothermal to isentropic conditions. However, to see how the burning-rate fluctuations are caused by the incident acoustic wave and then how they determine the reflected wave, across the entropy layer, it is easier to use expansions appropriate to the various regions.

To describe the acoustic waves, we choose for ℓ the wavelength

$$\ell_a = a_\infty/\omega \gg \ell_c/m_\infty \gg \ell_c \tag{38}$$

where $\ell_c = \lambda/c_p\rho_\infty v_\infty$ is, to within a factor ρ_c/ρ_∞, the unit of length

used in the combustion zone. We have used the restriction (2), which now reads

$$\omega << v_\infty/\ell_c \, , \tag{39}$$

and the fact that the Mach number m_∞ is small to obtain these extreme inequalities. It follows that the dimensionless ω is 1 but that κ_ℓ and λ_ℓ are small so that

$$p = Ae^{ix_a} + Be^{-ix_a} \, , \quad T = (\gamma-1)p/\gamma \, , \tag{40}$$

$$\rho = p/\gamma \, , \quad v = (-Ae^{ix_a} + Be^{-ix_a})/\gamma \tag{41}$$

where x_a is distance measured in units of ℓ_a. Here A and $B(m_\infty)$ are the amplitudes of the pressure in the incident and reflected waves, respectively, which would be dissipated over a distance

$$L_a = m_\infty \ell_a/\lambda_a >> \ell_a/m_\infty >> \ell_a \quad (\lambda_a \text{ is } \lambda_\ell \text{ for } \ell = \ell_a) \, . \tag{42}$$

For the theory to be applicable L_a must be large compared to an overall dimension but, since the latter is normally comparable to ℓ_a, that provides no restriction in practice. The goal is to determine B when A is given, so as to calculate the reflection coefficient $|B/A|$.

On the acoustic scale the whole combustion field is represented by a single point, which we may take to be $x_a = 0$. The temperature disturbance there is zero, which clearly contradicts the isentropic requirement (40b) of the acoustic waves when there is a pressure disturbance. A layer is needed to change the entropy from its constant value in the acoustic field as $x_a \to 0$. Now, in the absence of dissipation, entropy is convected

with the fluid; to describe these waves, we choose for ℓ the wavelength

$$\ell_e = v_\infty/\omega = m_\infty \ell_a \ll \ell_a \ . \tag{43}$$

The dimensionless frequency ω equals m_∞ and hence is small. We also find that κ_ℓ and λ_ℓ are still small, their dissipative effect being felt over a distance

$$L_e = m_\infty \ell_e/\lambda_e \gg \ell_e \qquad (\lambda_e \text{ is } \lambda_\ell \text{ for } \ell = \ell_e) \ . \tag{44}$$

If L_e is to be small compared to ℓ_a, we must have

$$\omega \gg m_\infty v_\infty/\ell_c \quad , \tag{45}$$

which now places a lower bound on the frequency for the theory to apply. The restriction is of no consequence in practice but, if it is not introduced, entropy effects extend into the far field and our simple picture is destroyed. The results must then be obtained from the full solution of equations (34,35).

We now come to the structure of the entropy layer, which the momentum equation shows to be isobaric, as might have been expected. The complete solution may therefore be written

$$T = (\gamma-1)p[1-\exp(-ix_e-X_e)]/\gamma \quad , \quad \rho = p[1+(\gamma-1)\exp(-ix_e-X_e)]/\gamma \ ,$$

$$v = m_\infty[-ipx_e/\gamma + (c-1)p] \tag{46}$$

to leading order, where x_e and X_e are distances measured in units of ℓ_e and L_e, respectively. Here the constants of integration have been determined so that

$$T = 0 \quad \text{and} \quad \rho + v/m_\infty = cp \tag{47}$$

at $x_e = X_e = 0$, as required by the analysis of the combustion regions: to $O(1)$ accuracy in θ the temperature is unaffected and $m = cd$ (in the notation of Secs. 2 and 3). Note that $v \to \infty$ as $x_e \to \infty$, a fact that makes a careful analysis of the entropy layer necessary, as we shall see shortly.

As $X_e \to \infty$ the solution (46) becomes isentropic, i.e. the relations (40b, 41a) are satisfied. A formal matching is obtained by taking m_∞ as the vanishingly small parameter, which is suggested by the transformation $x_a = m_\infty x_e$. When applied to v the matching shows that

$$B = A(1+m_\infty\alpha) \quad \text{with} \quad \alpha = 2\gamma(c-1) \tag{48}$$

correct to first order in m_∞. The reflected wave is thereby determined in terms of the incident wave. [Use of m_∞ as small parameter makes the inequalities (39) and (45) become $O(m_\infty^3) << \omega << O(m_\infty^2)$.]

Usually such reflection problems are discussed in terms of the acoustic admittance. In the present context Williams (1973) equates the value of v/p as $x_a \to 0$ to its value in the combustion region, which happens to give the correct result (48). It would be more logical to equate it to the value at the edge of the entropy layer, but that is impossible since $v \to \infty$ as $x_e \to \infty$. [The difficulty is not a creature of our analysis: the continuity equation (31) requires v to become unbounded whenever ρ does not vanish at the edge of the layer.] Only proper matching of the entropy layer resolves the delemna.

5. Amplification

Since A is the amplitude of the incident wave and B is that of the reflected wave, there is amplification for small values of m_∞ if and only if

$$\mathrm{Re}(\alpha) > 0 \quad \text{i.e.} \quad \mathrm{Re}(c) > 1 \quad . \tag{49}$$

We shall now examine the two cases of solid pyrolysis in Sec. 2 and the two cases of liquid evaporation in Sec. 3 for this instability.

The response (17) for solid pyrolysis shows what Re(c) is never greater than 1/2, its value for $\tilde{\omega} = 0$, so that there is no instability. The conclusion is changed by giving D a pressure dependence p^n with $n > 2$ since then c is replaced by nc, whose maximum real part exceeds 1; but in practice n is never that large.

On the other hand the response (21) shows that

$$\mathrm{Re}(c) = 1/2 + [2P + 4 - (Q+P+2)^2]\omega^2/8 + O(\omega^4) \tag{50}$$

for small ω, which increases initially with ω for a band of values around

$$Q = -(P+2) \quad , \tag{51}$$

suggesting that the inequality (49) may be satisfied for some ω when P and Q are chosen appropriately. This is confirmed by graphs of Re(c) versus ω for several values of P in Fig. 3, where the value (51) has been taken for Q to ensure the fastest rate of initial increase.

Turning now to the evaporating liquid, we find that the response (28) gives

$$\mathrm{Re}(c) = 1 - \frac{2+\tilde{\omega}^2 q(p+q)}{1+\tilde{\omega}^2 q^2} . \tag{52}$$

For instability q must be negative but greater than $-p$; then all frequencies

$$\tilde{\omega} > \sqrt{-2/q(p+q)} \tag{53}$$

are amplified. On the other hand, $\mathrm{Re}(c)$ for the response (29) behaves like $-P\sqrt{\omega}/\sqrt{2}(2+Q)$ as $\omega \to \infty$, so that 1 is certainly exceeded for $Q < -2$ whatever the value of P. Figure 4 shows graphs for $P = 1$ and various values about $Q = -2$.

The absence of amplification at all frequencies $\tilde{\omega}$ for a pair of values p, q does not necessarily mean that stable burning will take place. There may be inherent instability. For solid pyrolysis, parameter values for which $p+q$ is negative are ruled out even though there is no amplification of acoustic waves. For liquid evaporation there is stable burning only when q is positive. Similar remarks apply to the cases governed by P and Q.

6. Stability of Anchored Flames

The flames considered in this chapter are anchored to the condensate in the sense that they are controlled by the manner in which the reactant is liberated at the surface (which in turn is affected by heat from the flame). The stability of certain anchored flames has therefore been covered already. Here we shall consider a stronger control in which there is no influence of the flame on the source, i.e. the reactant is supplied at constant temperature T_s, any heat conducted back from the flame being instantly removed. The flamholder of Sec. II.4 is an example, if there

is temperature control at the porous plug (by means of heating/cooling coils) and the flow rate M is maintained constant. While stability is the main interest, we shall also consider the effect of an impinging acoustic wave, i.e. oscillations in D.

References

Denison, M.R. and Baum, E. 1961, A simplified model of unstable burning in solid propellants, ARS J. **31**, 1112-

Williams, F.A. 1973, Quasi-steady gas-phase flame theory in unsteady burning of a homogeneous solid propellant, AIAA J. **11**, 1328-1330.

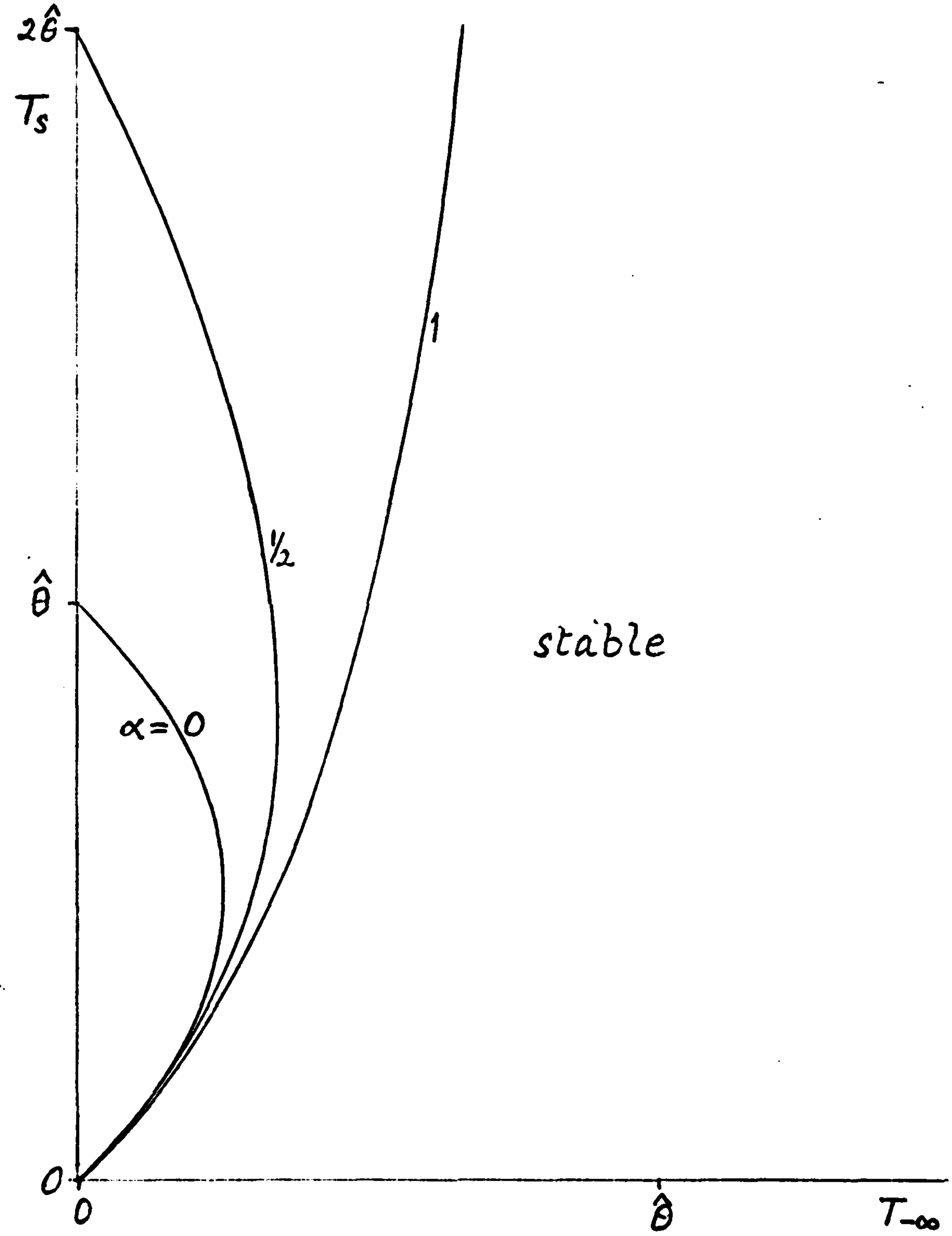

Fig. 1: Stability boundaries for solid pyrolysis; ω small.

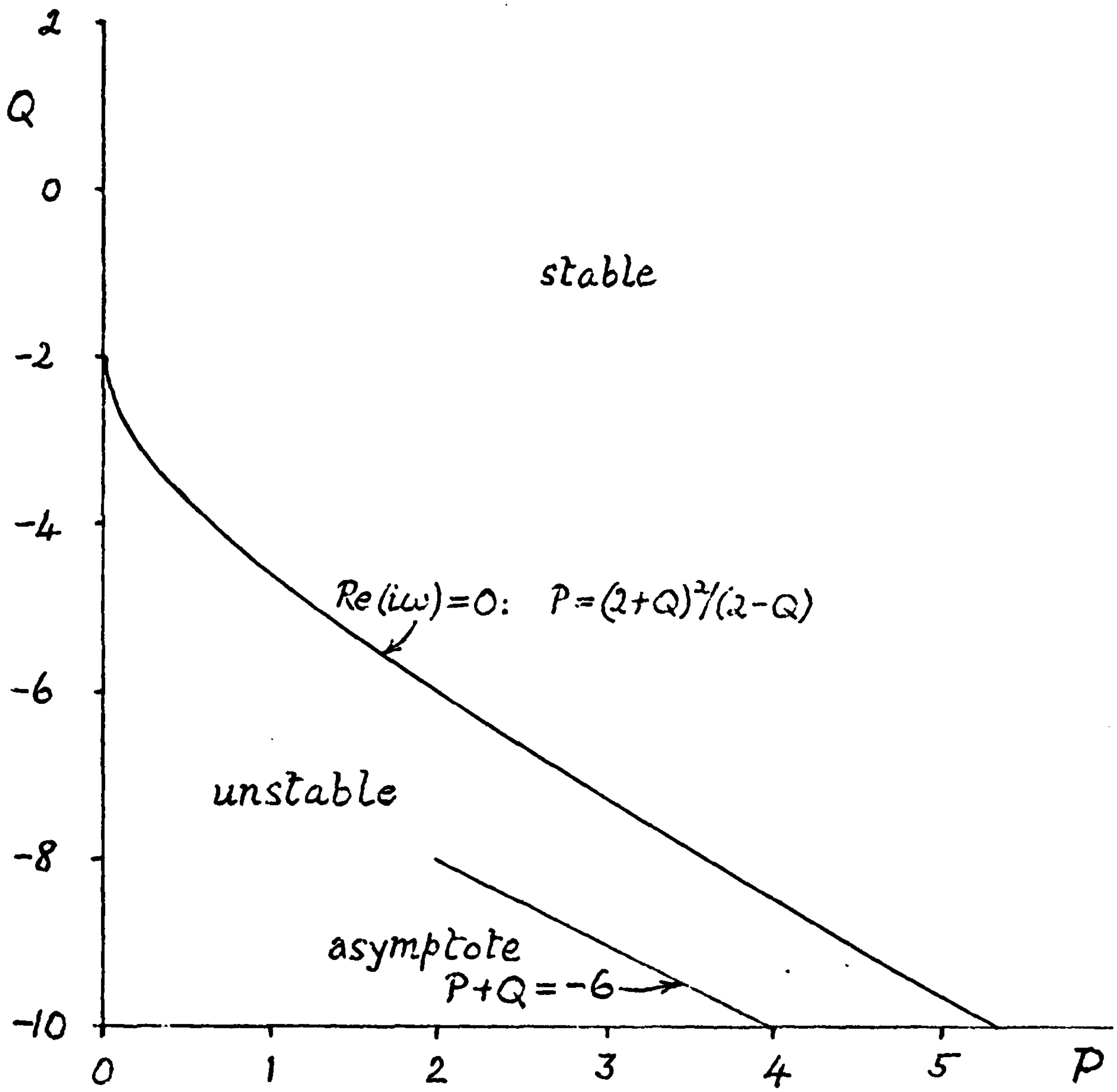

Fig 2: Stability boundary for solid pyrolysis

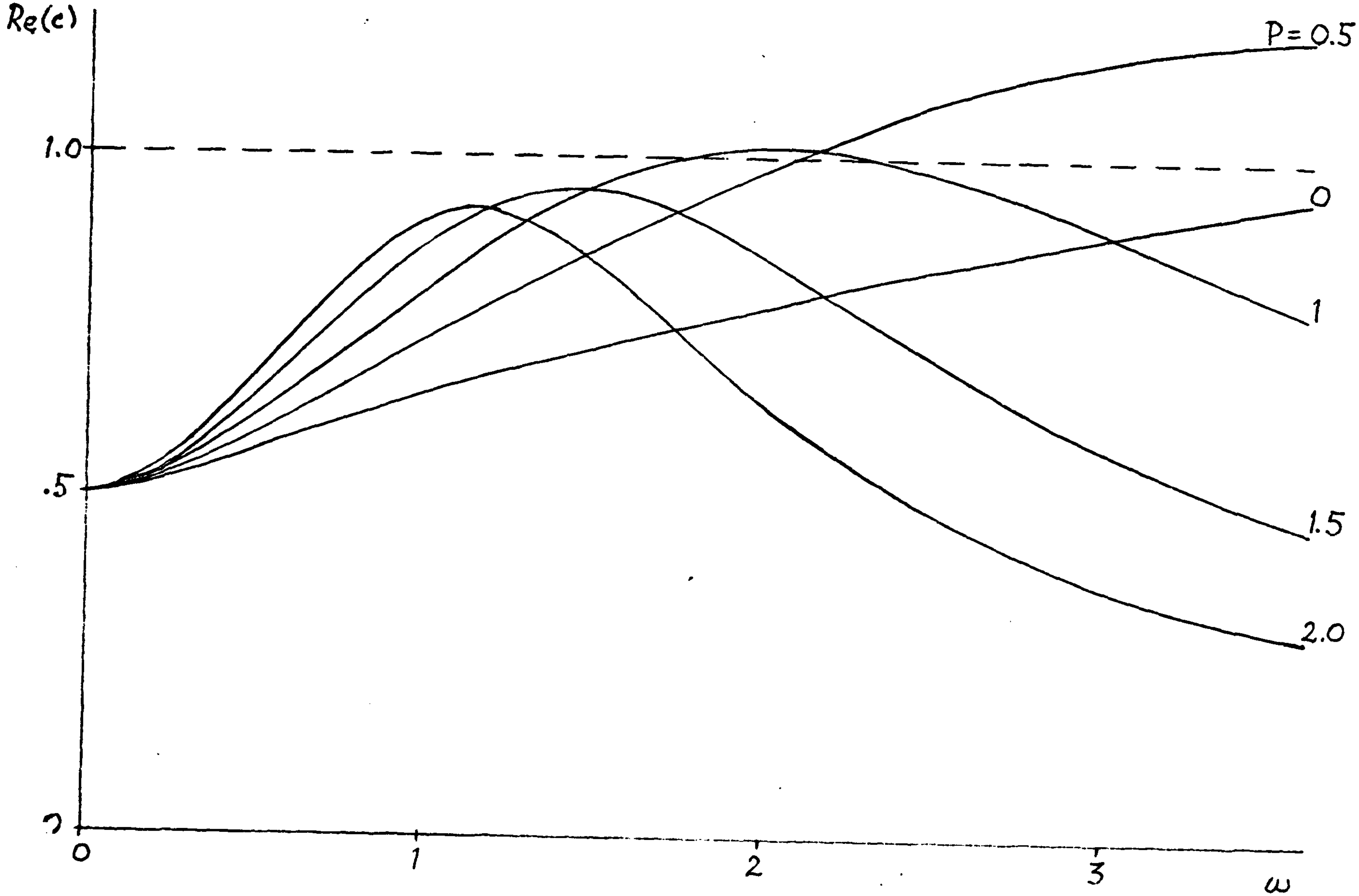

Fig. 3a: Amplification for solid pyrolysis: $Q = -(P+2)$, small P.

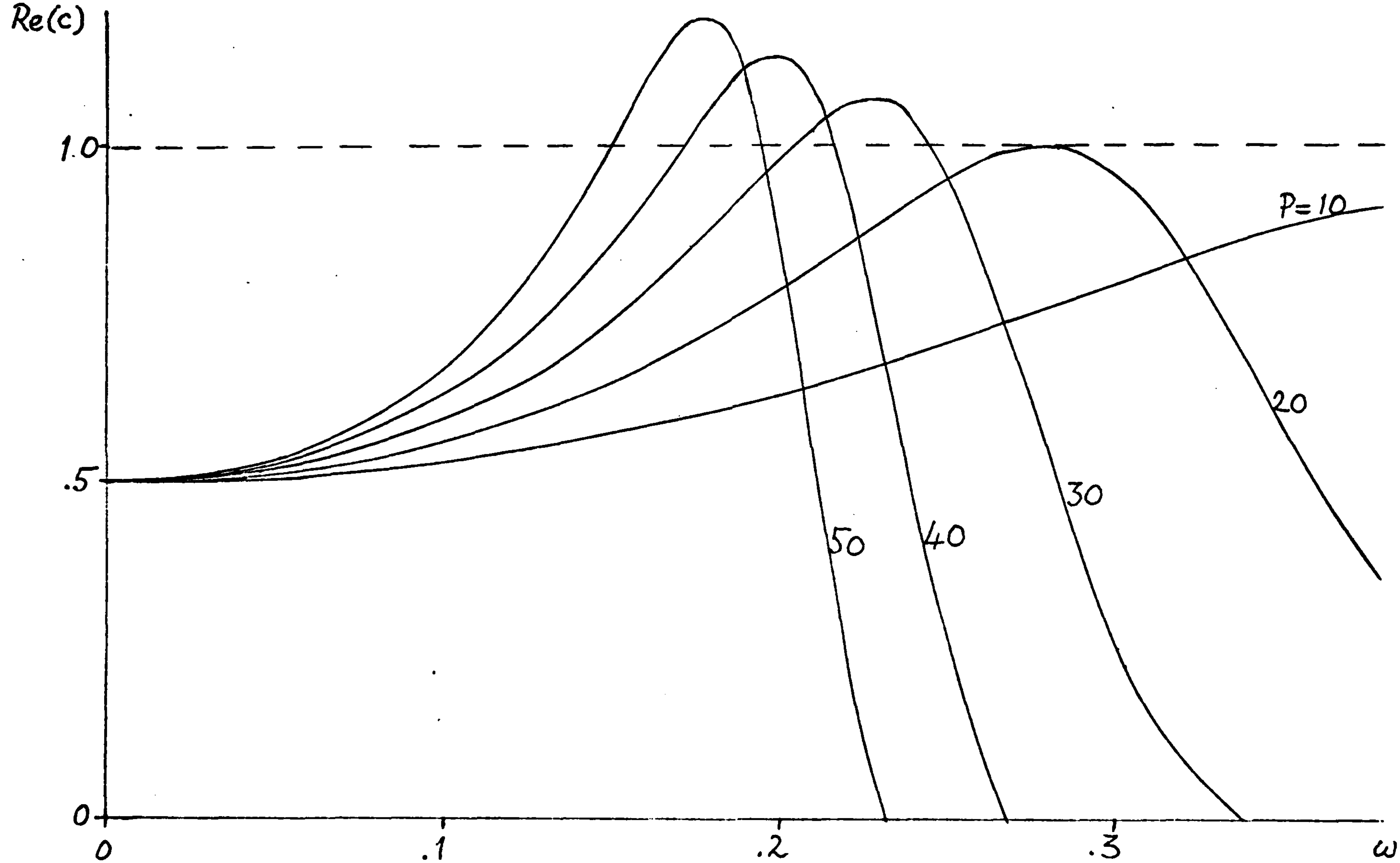

Fig. 3b: Amplification for solid pyrolysis: $Q=-(P+2)$, P large.

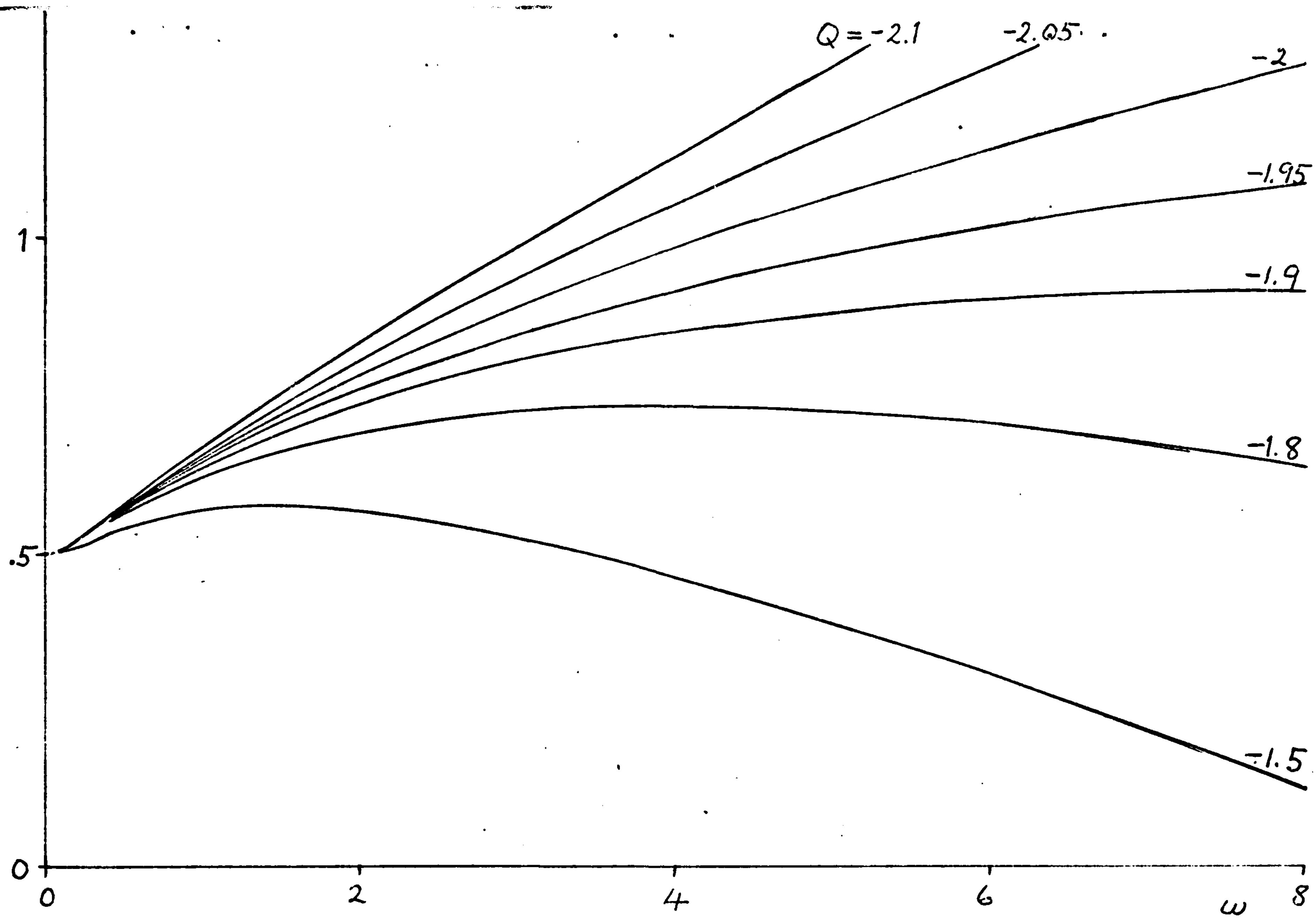

Fig. 4: Amplification for liquid evaporation: $P = 1$.

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The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

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Unsteady burning, linear condensate, solid pyrolysis, liquid evaporation, impinging acoustic wave, stability, amplification, anchored flames.

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This report is Chapter V of the twelve in a forthcoming research monograph on the mathematical theory of laminar combustion. The unsteady burning of a linear condensate is discussed due, for example, to an impinging acoustic wave. Stability of steady states is also considered.

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